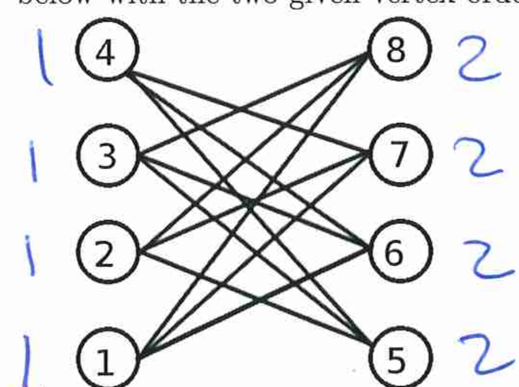
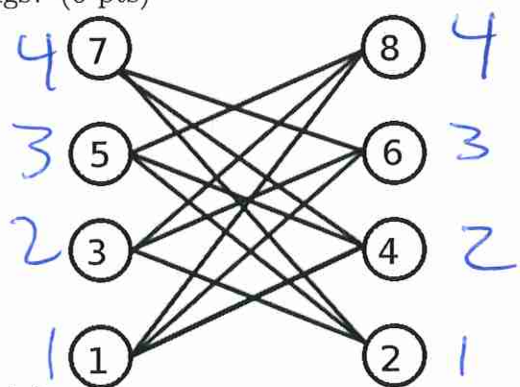
Name: Mr. Solutions RCS ID: __________ @rpi.edu

Graph Theory Quiz 4 (14 June 2019)
Open book, open notes, open neighbor.

1. Using the greedy coloring algorithm, how many colors will result on the graph G below with the two given vertex orderings? (6 pts)



a.)



b.)

$\boxed{2}$ $\boxed{4}$

2. Tightly bound the possible chromatic numbers of G, $\chi(G)$. Justify your response using the bounds discussed in class.

G is bipartite

$\boxed{\chi(G) = 2}$

3. Is G color-critical? Justify your response.

$\boxed{\text{No}}$ We've bounded $\chi(G)=2$

In order to have $H = G - e$ s.t.

$\chi(H) = 1$, G would need to have only a single edge

4. Recall that C_n is color-critical for $n = odd$. Show that any graph G is k-color-critical for $\chi(G) = k = 3$ if and only if G is an odd cycle.

If G is odd cycle $\Rightarrow$ G is color-critical and $\chi(G) = 3$

- We demonstrated that all odd cycles have chromatic number of 3 in class with a greedy coloring argument.
- Odd cycles are color-critical, as removing any edge creates a path, colorable w/ 2 colors

G is color-critical and $\chi(G) = 3 \Rightarrow G$ is odd cycle

- Color-critical implies removing some e will decrease the chromatic number
- G is not acyclic nor bipartite (as then $\chi(G) = 2$)

so G contains cycles, at least one of which is odd

- Consider removing a hypothetical e not in an odd cycle; as the odd cycle remains, the chromatic number is still 3
- Hence, any such e must be in an odd cycle and there can only be one odd cycle

$\Rightarrow$ G is an odd cycle $\square$